ECE317 : Feedback and Control

Lecture:
Modeling of electrical & mechanical systems

Dr. Richard Tymerski
Dept. of Electrical and Computer Engineering
Portland State University

Course roadmap

Modeling

✓ Laplace transform

Transfer function

Block Diagram
Linearization

➤ Models for systems
- electrical
- mechanical
- example system

Analysis

Stability
- Pole locations
- Routh-Hurwitz

Time response
- Transient
- Steady state (error)

Frequency response
- Bode plot

Design

Design specs

Frequency domain

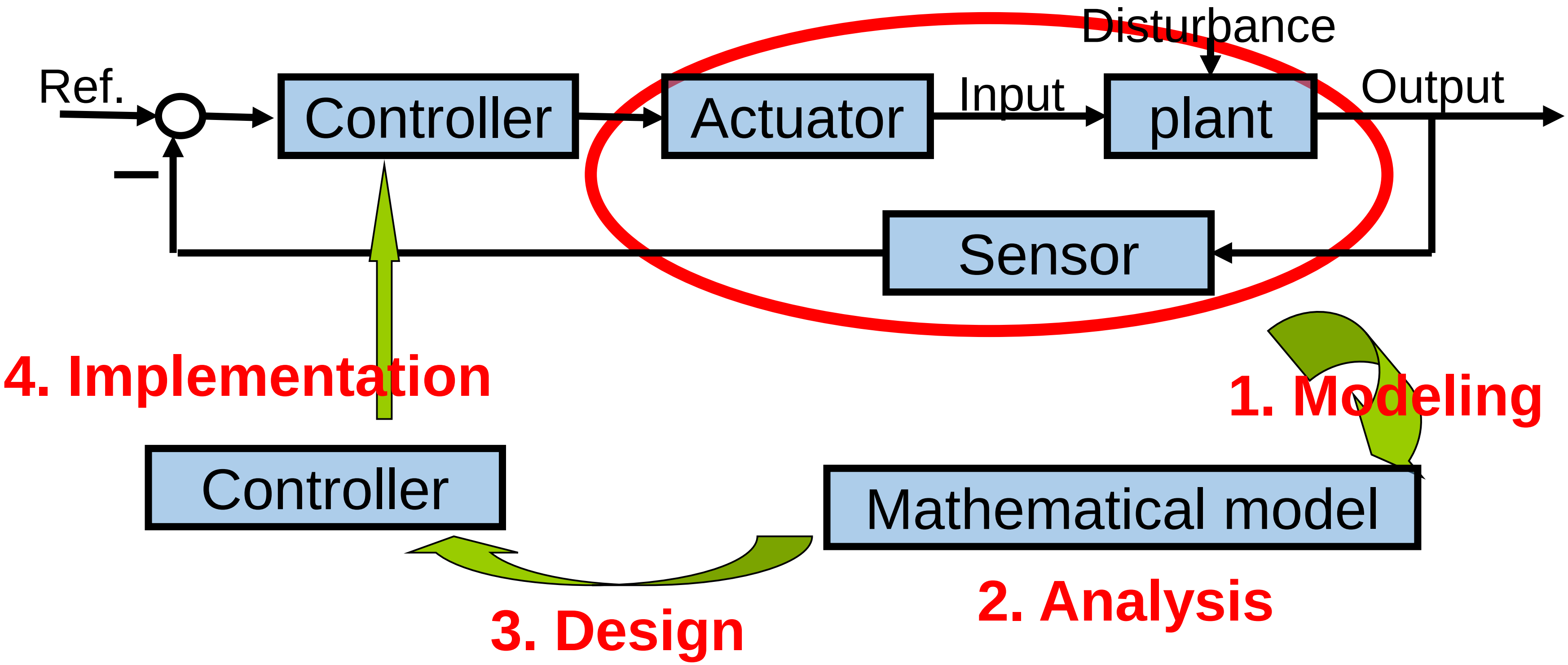
Bode plot

Compensation

Design examples

Matlab & PECS simulations & laboratories

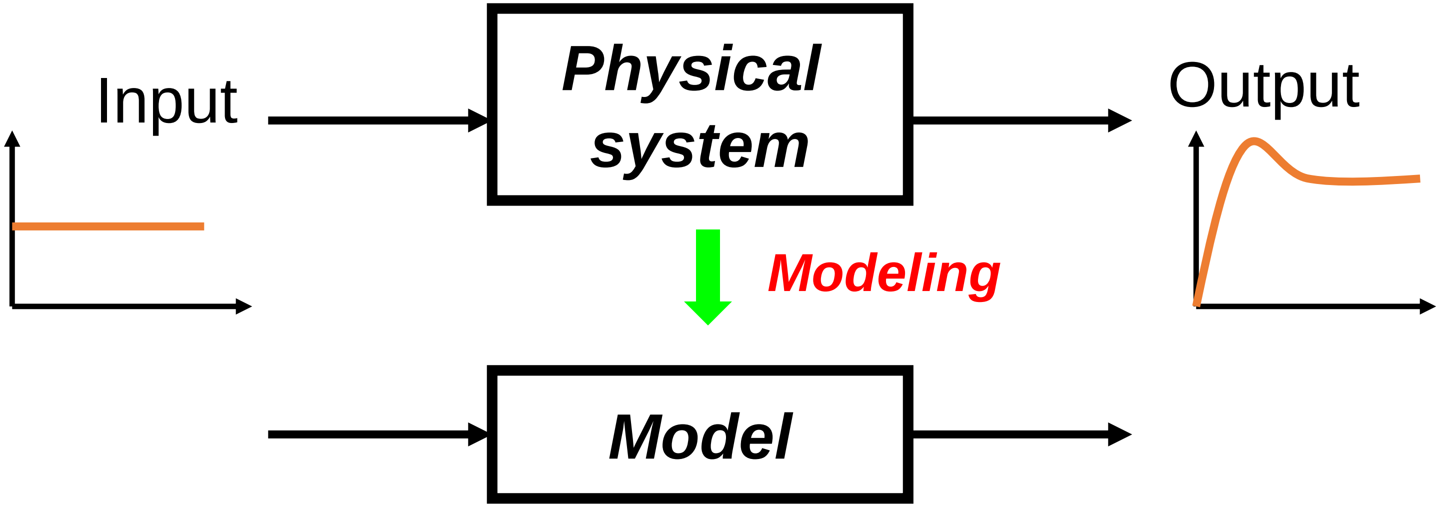
Controller design process (review)



- What is the “mathematical model”?
- Transfer function
- Modeling of electrical & mechanical systems

Mathematical model

- Representation of the input-output (signal) relation of a physical system



- A model is used for the **analysis** and **design** of control systems.

Important remarks on models

- Modeling is the most important and difficult task in control system design.
- No mathematical model exactly represents a physical system.

$$\text{Math model} \neq \text{Physical system}$$
$$\text{Math model} \approx \text{Physical system}$$

- Do not confuse models with physical systems!
- In this course, we may use the term "system" to mean a mathematical model.

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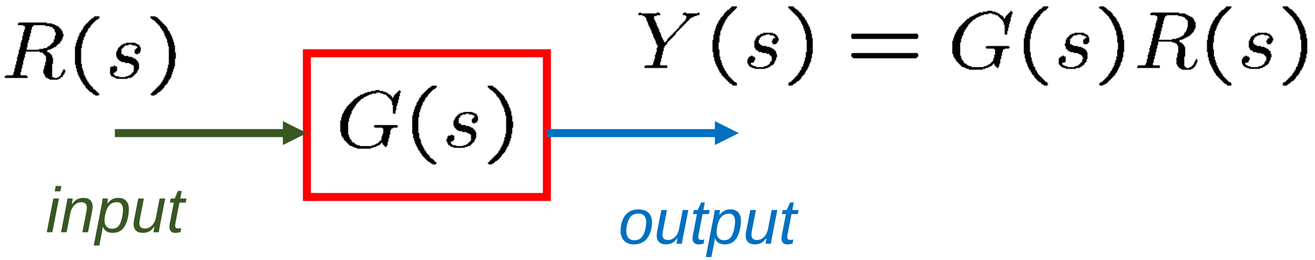
Transfer function

- A transfer function is defined by

$$G(s) := \frac{Y(s)}{R(s)}$$

Laplace transform of system output

Laplace transform of system input



$R(s)$

$G(s)$

$Y(s) = G(s)R(s)$

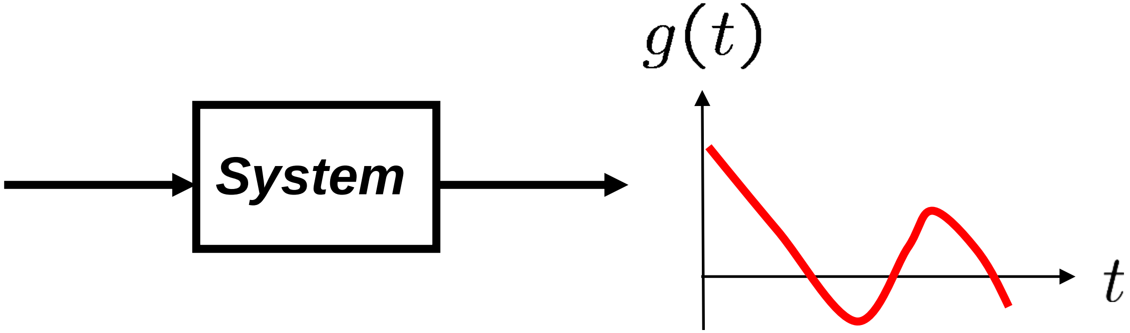
- A system is assumed to be at rest. (zero initial condition)
- Transfer function is a generalization of “gain” concept.

Impulse response

- Suppose that *r(t)* is the unit impulse function and system is at rest.

$$r(t) = \delta(t)$$

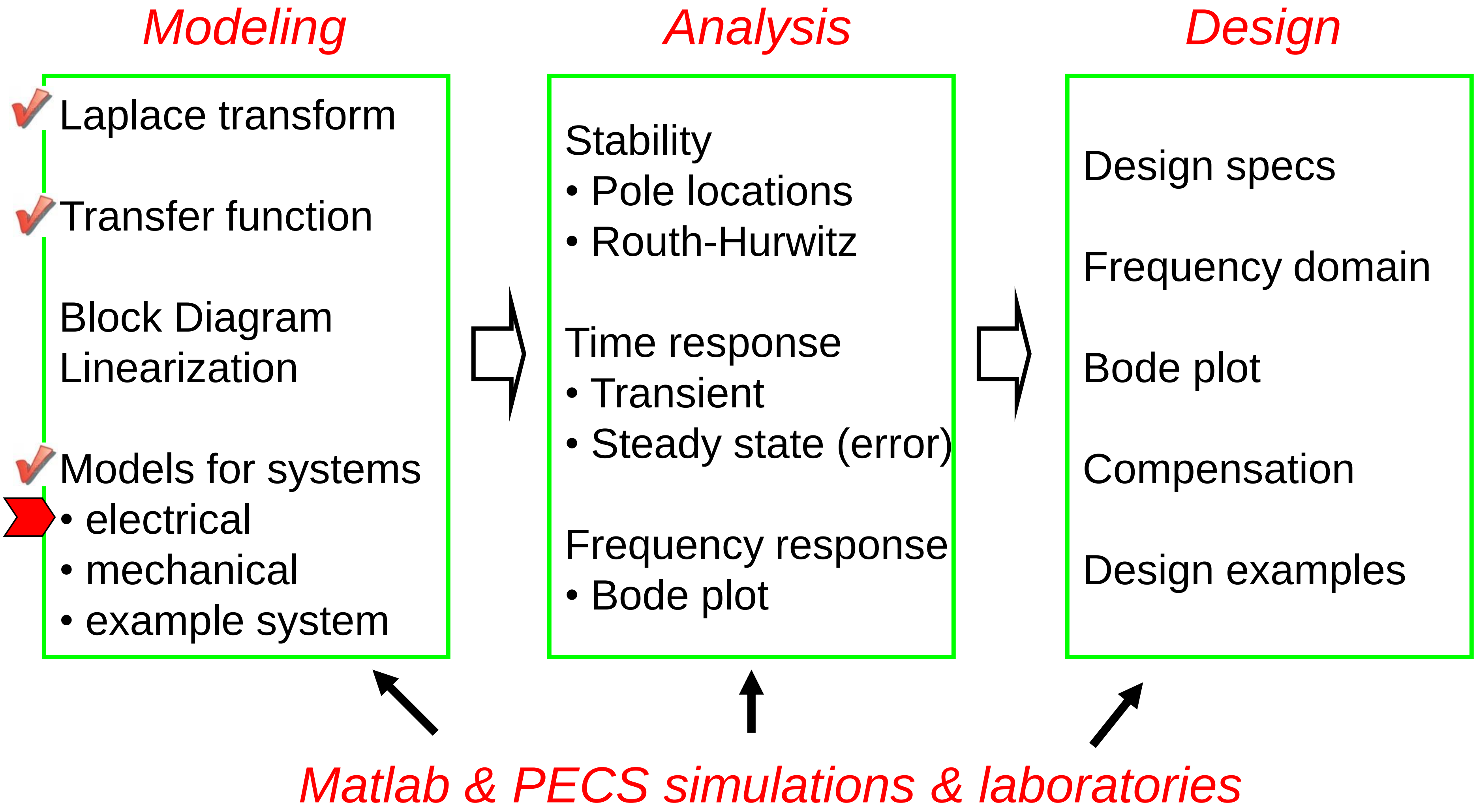
$$R(s) = 1$$



- The output *g(t)* for the unit impulse input is called *unit impulse response*.
- Since *R(s)=1*, the transfer function can also be defined as the Laplace transform of impulse response:

$$G(s) := \mathcal{L}\{g(t)\}$$

Course roadmap



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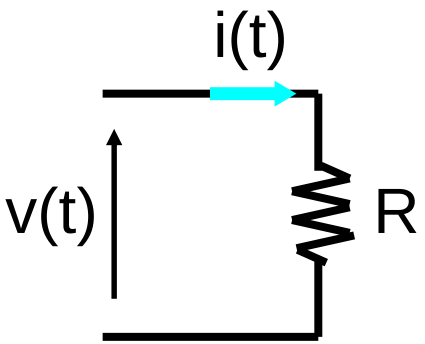
Models of electrical elements

Resistance

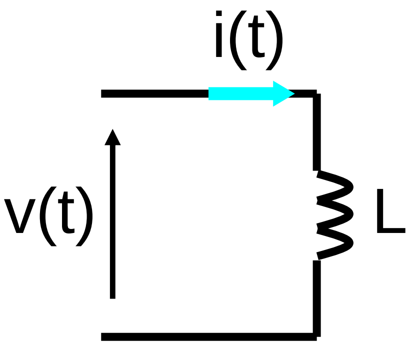


$$v(t) = Ri(t)$$

Laplace transform

$$\frac{V(s)}{I(s)} = \underline{R}$$

Inductance



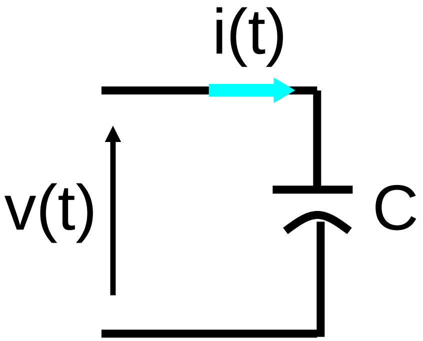
$$v(t) = L\frac{di(t)}{dt}$$

$$(i(0) = 0)$$

$$\frac{V(s)}{I(s)} = \underline{sL}$$

Impedance

Capacitance

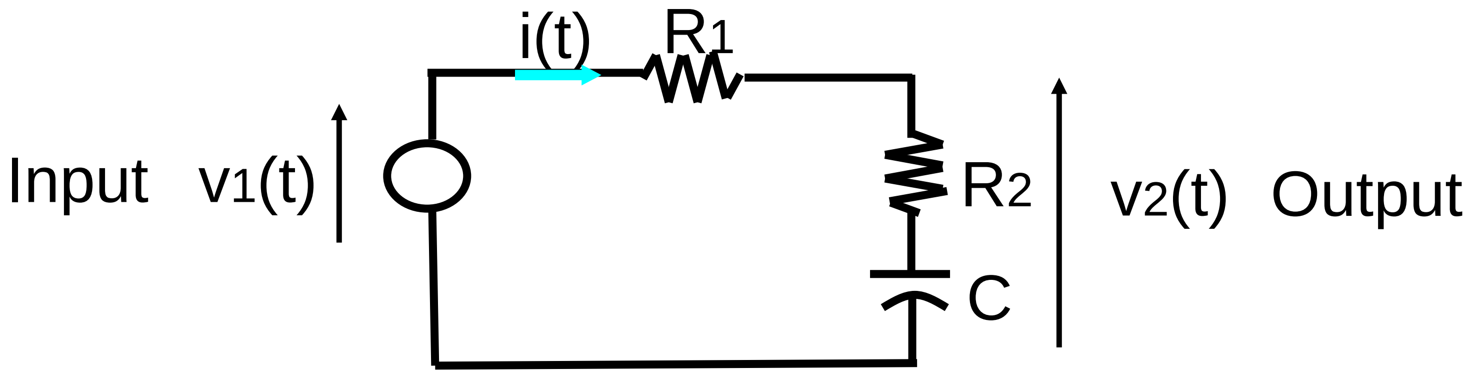


$$i(t) = C\frac{dv(t)}{dt}$$

$$(v(0) = 0)$$

$$\frac{V(s)}{I(s)} = \underline{\frac{1}{sC}}$$

Modeling example



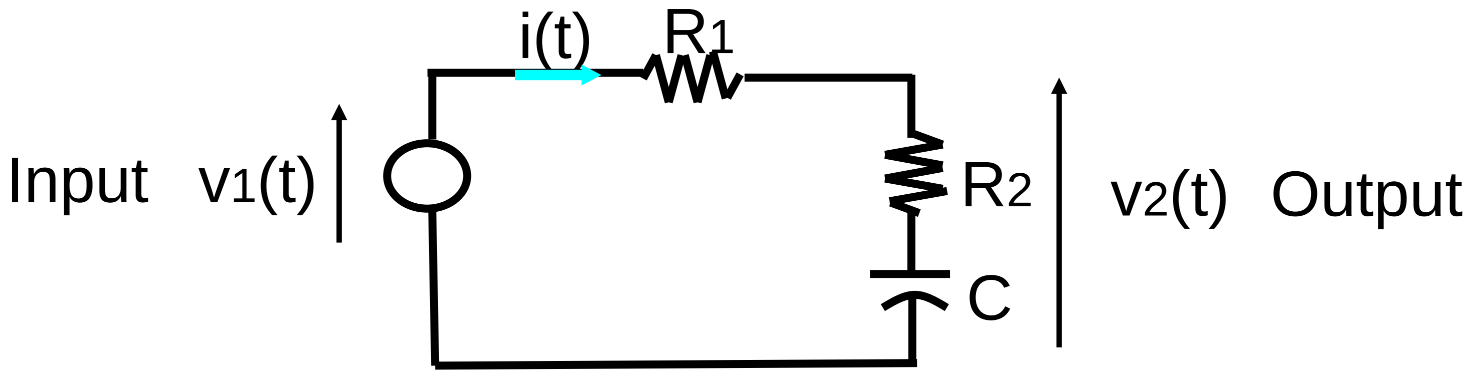
- Kirchhoff voltage law (with zero initial conditions)

$$\begin{aligned} v_1(t) &= (R_1 + R_2)i(t) + \tfrac{1}{C}\int_0^t i(\tau)d\tau \\ v_2(t) &= R_2 i(t) + \tfrac{1}{C}\int_0^t i(\tau)d\tau \end{aligned}$$

- By Laplace transform,

$$\begin{aligned} V_1(s) &= (R_1 + R_2)I(s) + \tfrac{1}{sC}I(s) \\ V_2(s) &= R_2 I(s) + \tfrac{1}{sC}I(s) \end{aligned}$$

Modeling example (cont'd)



- Transfer function

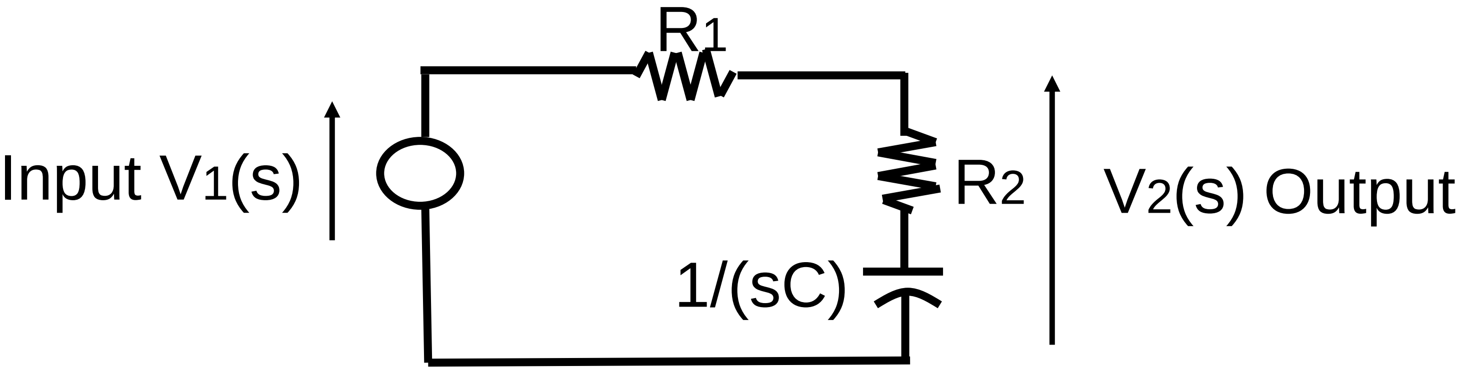
$$G(s) = \frac{V_2(s)}{V_1(s)} = \frac{R_2 + \frac{1}{sC}}{(R_1+R_2) + \frac{1}{sC}}$$

$$= \frac{R_2Cs+1}{(R_1+R_2)Cs+1}$$ (first-order system)

- If output is *i(t)*, then ….

Modeling example (cont'd)

- Impedance method
  - Replace electrical elements with impedances.
  - Deal with impedances as if they were resistances.



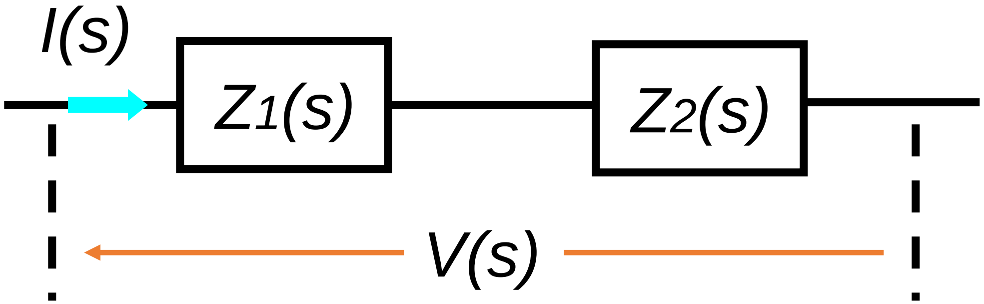
$$G(s)=\frac{V_2(s)}{V_1(s)}=\frac{\text{(Impedance for output)}}{\text{(Total impedance)}}=\frac{R_2+\frac{1}{sC}}{R_1+R_2+\frac{1}{sC}}$$

Impedance computation

- Series connection

$Z_i(s)$: impedance

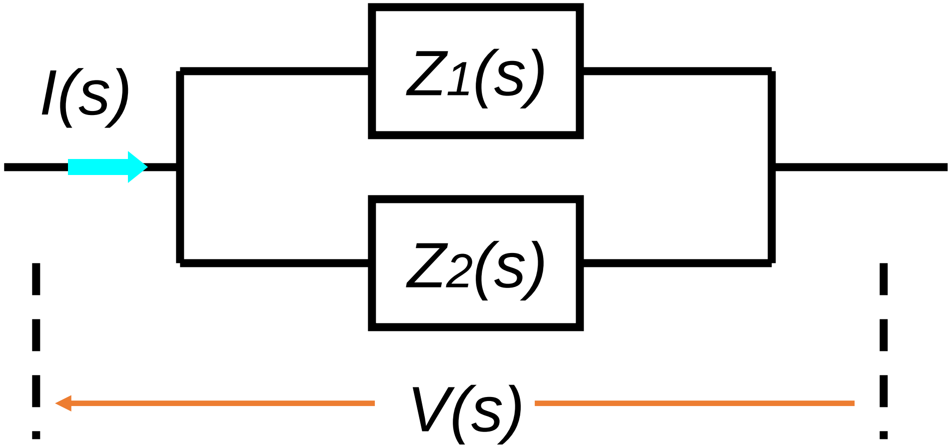
$$Z(s) = Z_1(s) + Z_2(s)$$



- Parallel connection

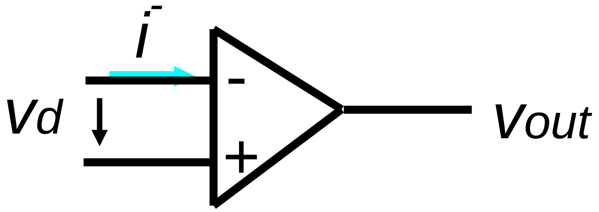
$$Z(s) = \frac{Z_1(s)Z_2(s)}{Z_1(s) + Z_2(s)}$$

$$\frac{1}{Z(s)} = \frac{1}{Z_1(s)} + \frac{1}{Z_2(s)}$$



Operational amplifier (op-amp)

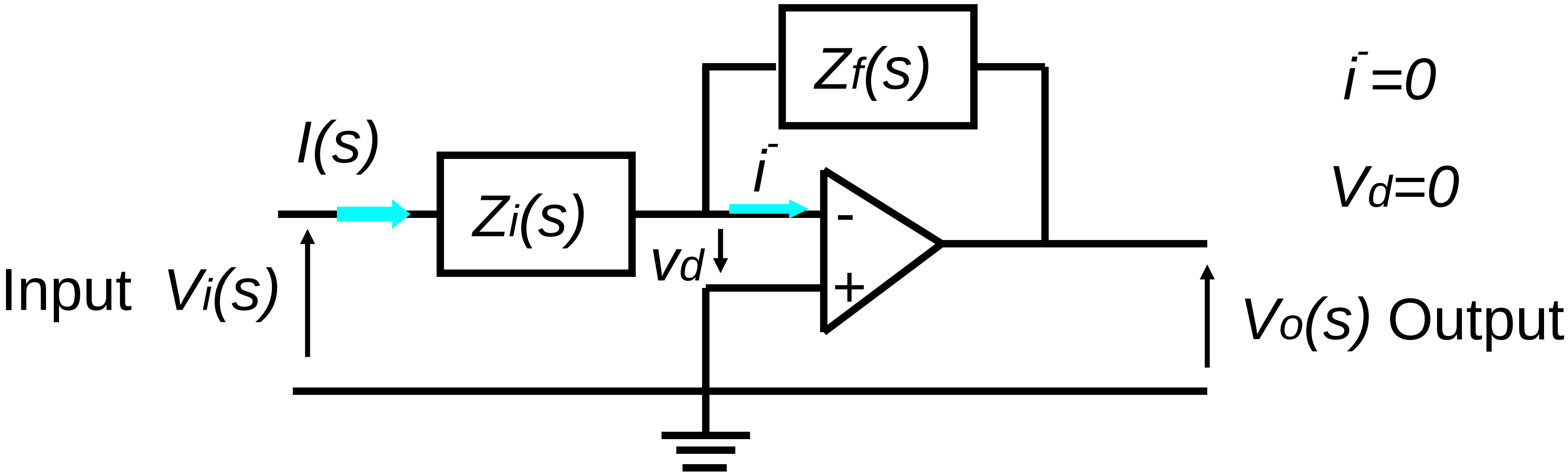
- Electronic voltage amplifier
- Basic building block of analog circuits, such as
  - Voltage summation (math operation)
  - Voltage integration
  - Various transfer functions (Signal conditioning, filtering)
- Ideal op-amp (does not exist, but is a good approximation of reality)



$i^- = 0$

$V_d = 0$

Modeling of op-amp



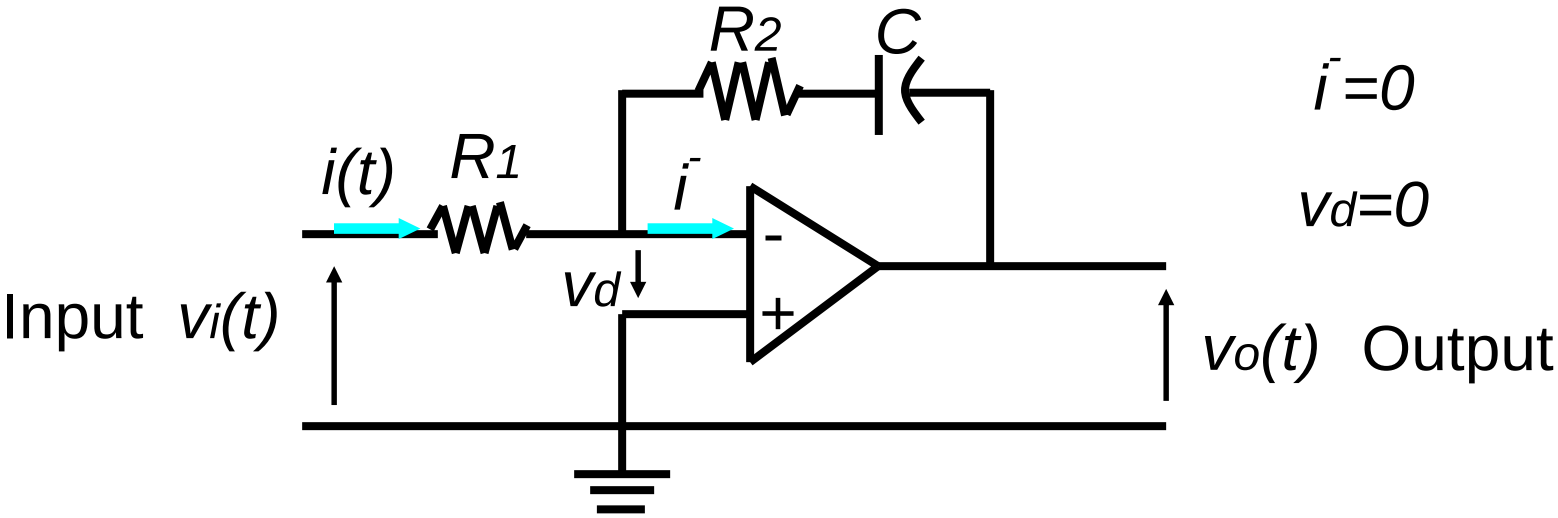
$i^- = 0$

$V_d = 0$

- Impedance *Z(s): V(s)=Z(s)I(s)*
- Transfer function of the above op amp:

$$G(s) = \frac{V_O(s)}{V_i(s)} = \frac{-Z_f(s)I(s)}{Z_i(s)I(s)} = -\frac{Z_f(s)}{Z_i(s)}$$

Modeling example: op-amp



$i^- = 0$

$v_d = 0$

- By the formula in previous two slides,

$$G(s) = \frac{V_o(s)}{V_i(s)} = \frac{-(R_2 + \frac{1}{sC})}{R_1} = -\frac{R_2Cs + 1}{R_1Cs}$$

(first-order system)

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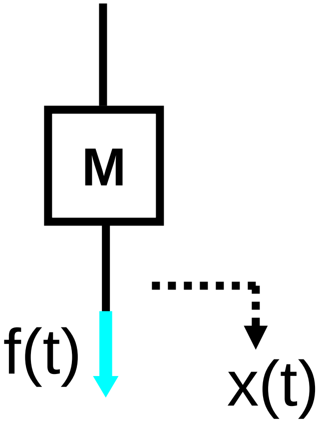
Translational mechanical elements

Mass

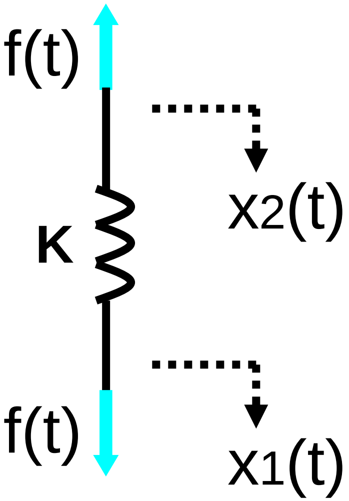


$$f(t) = Mx''(t)$$

$$\Downarrow \quad \begin{pmatrix} x(0) = 0 \\ \dot{x}(0) = 0 \end{pmatrix}$$

$$F(s) = Ms^2X(s)$$

Spring

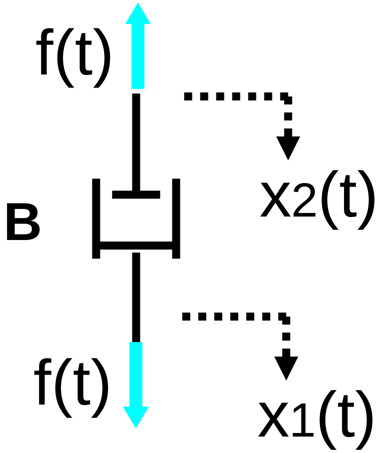


$$f(t) = K(x_1(t) - x_2(t))$$

$$\Downarrow$$

$$F(s) = K(X_1(s) - X_2(s))$$

Damper



$$f(t) = B(x_1'(t) - x_2'(t))$$

$$\Downarrow \quad \begin{pmatrix} x_1(0) = 0 \\ x_2(0) = 0 \end{pmatrix}$$

$$F(s) = Bs(X_1(s) - X_2(s))$$

Summary

- Modeling
  - Modeling is an important task!
  - Mathematical model
  - Transfer function
  - Modeling of electrical & mechanical systems
- Next lecture, block diagram reduction